MOISTURE TRANSFER IN A CLOSED SYSTEM WITH AN INTERNAL HEAT SOURCE

V. V. Puchkovskii

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The author examines moisture transfer between a solid containing an internal heat source and a liquid in a system closed in the moisture-transfer sense. The influence of the difference in sorption characteristics between solid and liquid on the redistribution of moisture is shown. Internal moisture transfer in high-voltage power transformers is considered as an example.

In solving problems involving heat and mass transfer with boundary conditions of the third kind, the values of the temperature and mass content of the medium are mostly assumed constant. In other cases, the variation of temperature or mass content is given as a function of time. Amont problems relating to a medium with variable parameters, one may distinguish those in which the process is local and takes place within a sharply defined volume, for example, heat and mass transfer between a solid and a liquid* placed in a vessel separating them from an infinite surrounding medium. Here we may consider, in general, cases where the vessel walls transmit neither heat nor mass, or only one of them. Then the solid-liquid system must be considered closed with respect to heat and mass transfer, or only one of them. To the conditions of heat and mass transfer in such problems we must, in general, add the fact that the amount of heat and mass in the system is constant.

One such problem is the approximate analysis of the internal moisture transfer in equipment such as high-voltage power transformers. Internal moisture transfer in a system of this type proceeds inside the tank between a solid, the hygroscopic insulator, mainly cardboard, and a liquid – mineral oil.

We limit the problem by assuming that the solid is a plate whose thickness is small in comparison with its length or width, or a group of such plates, or a group of thin-walled cylinders of sufficiently great length and radius, the boundary conditions being represented by Bi = Bi = ∞. We then find the steady temperature and humidity distribution, assuming that there is a distributed heat source inside the plate (dielectric losses) that depends both on temperature and humidity. Let the system of differential equations have the form

$$a_0 \frac{\partial^2 t(x)}{\partial x^2} + \frac{P_0}{c\gamma} \exp\{a[t(x) - t_{\mathrm{p}}]\}[1 + bu(x)] = 0, \tag{1}$$

$$\frac{\partial^2 u(x)}{\partial x^2} + \delta \frac{\partial^2 t(x)}{\partial x^2} = 0. \tag{2}$$

Integrating twice and determining the constants of integration from the symmetry condition, from (2) we get

$$u(x) = u_{\mathrm{e}} - \delta[t(x) - t_{\mathrm{e}}]. \tag{3}$$

Substituting the humidity value found into (1) and going over to the relative coordinate, we obtain

$$\frac{\partial^2 t(x_*)}{\partial x_*^2} + \frac{P_0 R^2}{\lambda} \exp\{a[t(x_*) - t_{\mathrm{p}}]\}[1 + b\{u_{\mathrm{e}} - \delta[t(x_*) - t_{\mathrm{e}}]\}] = 0, \tag{4}$$

or

$$\frac{\partial^2 t(x_*)}{\partial x_*^2} = -\frac{P_0 R^2}{\lambda} \varphi[t(x_*)]. \tag{5}$$

We note that $P_0R^2/\lambda = T_y$ has the dimension of temperature and is equal to the excess above the boundary surface temperature attained in the middle plane of the plate when a constant heat source of specific power P_0 is not distributed throughout the volume of the solid, but concentrated in an infinitely thin film coinciding with the middle plane of the plate. Note, moreover, that for the assumed dependence of the power of the internal heat source on temperature and humidity the function $\varphi[t(x_*)]$ has two singular points: a maximum at $t_1 = \{a[1 + bu_e + b\delta t_e] - b\delta\}/ab\delta$ and a salient point (complete dehydration) at $t_2 = t_e + u_e/\delta$.

*Or a gas or gas mixture.

The temperature difference between these two points is then

$$t_2 - t_1 = -\mathrm{Di}/b\,\delta. \tag{6}$$

Here the dynamic parameter Di [1] of the internal heat source takes the form

$$\mathrm{Di} = \left(b\,\delta - \frac{\partial P(t,\ u)}{\partial t}\ \frac{1}{P(t,\ u)}\right)\left[\frac{\partial P(t,\ u)}{\partial t}\ \frac{1}{P(t,\ u)}\right]^{-1} = \frac{b\,\delta - a}{a}\ . \tag{7}$$

We find the approximate temperature distribution over the thickness of the plate by dividing one half of the plate into strips for which the right side of (5), determined from the mean value $\bar{\varphi}[t(x_*)]$, may be assumed constant. Expansion in power series leads to

$$\varphi\,[t\,(x_*)] = \mathrm{const}\left[1 + \sum_n \left(1 - \frac{n!\ k}{a}\right) a^n t^n\,(x_*)\right], \tag{8}$$

where $k = b\delta/(1 + bu_e + b\delta t_e)$. It can be shown that the determination from the mean is limited to a maximum error for terms of the series of $1 \cdot 10^{-2}$ when $\Delta t \leq 0.2000\bar{t}$ and $5 \cdot 10^{-2}$ when $\Delta t \leq 0.4472\bar{t}$. Forming the difference equations

$$\Delta\,t_{n;\ n+1} = \frac{1}{2}\,T_y\,\bar{\varphi}\,[t\,(x_*)]\,(x_{*n}^2 - x_{*n+1}^2) \tag{9}$$

and assigning the interval Δt for the chosen value of T_y, we find the temperature distribution. Then, using (3), we determine the moisture content distribution over the thickness of the plate, and then its mean value $\bar{u}$. Obviously, displacement of part of the moisture from the plate into the surrounding liquid (or gas) will change the equilibrium conditions at the boundary, increasing the equilibrium moisture content to u'_{es} (Fig. 1).

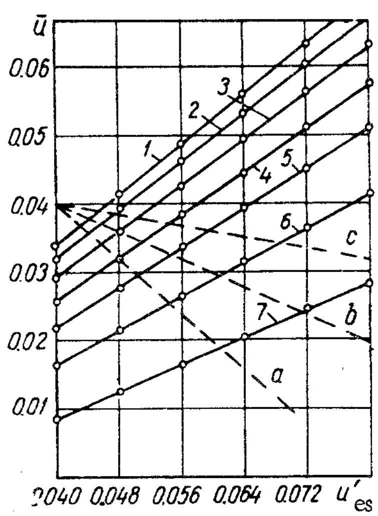

Fig. 1. Graphs for determining the state of moisture equilibrium between solid and liquid from the characteristics $\bar{u} = f_1(u'_{es})$ at temperatures from 10 to 70°C (1-7), and $f_2(u'_{es})$ for $\psi = 1$(a), 2(b) and 3(c)

Because the system is closed with respect to moisture transfer and the total amount of moisture is constant, the mean moisture content of the plate material may be determined as

$$\bar{u} = [(1 + \psi)\,u_{es} - u'_{es}]/\psi = f_2\,(u'_{es}). \tag{10}$$

Here the parameter $\psi = \dfrac{u_{es}}{u_{el}} \times \dfrac{G_s}{G_l} = \dfrac{w_s}{w_l}$ is used in the sense of a characteristic of the moisture distribution between the solid and the liquid for the initial moisture equilibrium at $T_y = 0$ (Fig. 1)(G_s and G_l define the solid-to-liquid mass ratio). The points of intersection of the curves derived from (10) with the characteristics $\bar{u} = f_1(u'_{es})$ give the possible states of moisture equilibrium at various liquid temperatures under the influence of an internal heat source.

Here we are considering cases where the heat source power is insufficient to have any appreciable influence on the liquid temperature. The liquid temperature may vary under the influence of heat transfer with the outside medium or due to sources in the system itself that do not take part in the moisture transfer. In transformers, in particular, such sources are losses in the conductors and in the magnetic circuit.

Figure 2 shows curves of the equilibrium and average moisture contents of the solid as a function of the liquid temperature obtained from the constructions in Fig. 1.

The fact that ψ is constant implies similarity of the moisture absorption characteristics for the solid and the liquid and the dependence of moisture equilibrium between them on temperature. These conditions cannot be satisfied in practice. Figure 3 shows a dependence, similar to that of Fig. 2, for the assumption that ψ depends linearly on temperature. Comparison of these relations with those for a constant value of $\psi = 1$ shows that an increase in ψ suppresses the escape of moisture from the plate, while a decrease in ψ leads to an increase in moisture transfer.

Our results can be used to analyze processes with a variable liquid temperature, if the rate of change is small enough and if $\dfrac{\partial t\,(x,\ \tau)}{\partial \tau} = \dfrac{\partial u\,(x,\ \tau)}{\partial \tau} = 0$ in the heat and mass transfer equations. In parcitular, such a process may also be quasi-steady, described by an harmonic function of time with a sufficiently large period.

For our two materials – cardboard and transformer oil – the ratio of equilibrium moisture contents depends both on the temperature and on the moisture contents. Relations for the equilibrium moisture contents of cardboard and oil have been given [2] for a number of temperatures. Using these, we can find the temperature dependence of $\bar{u}$ and u'_{es} for G_0/G_c and plot the periodic variation of the moisture content of the liquid (oil) during a period of the temperature

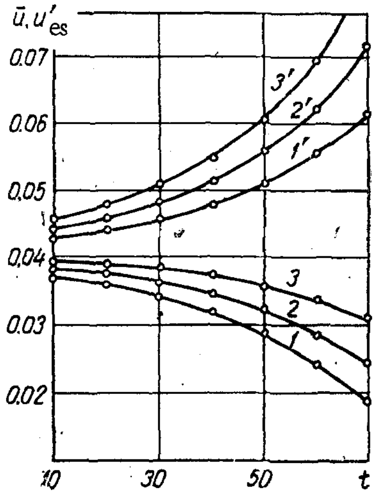

Fig. 2. Variation of mean (1, 2, 3) and equilibrium (1', 2', 3') moisture content of solid with increase liquid temperature for ψ = 1, 2, and 3, respectively.

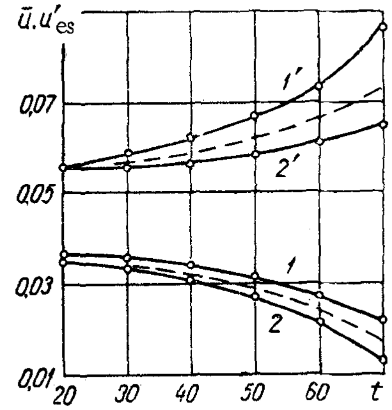

Fig. 3. Variation of mean (1, 2) and equilibrium (1', 2') moisture content of solid with increase in liquid temperature for $\psi_{1,2}$ = 1.00 ± 0.01 (broken line: ψ = 1.0).

variation. Considerable hydration of the oil in the "hot" part of the period has been fairly well observed; it increases as G_c and G_0 approach. By observing power transformers in operation [3] and comparing the variation of the mean daily temperature of the upper layers of the oil with its moisture content, we obtain a picture of the annual variation (Fig. 4).

The nature of the curve of periodic variation of the moisture content of transformer oil and its position relative to the curve of temperature variation are in complete agreement with our plot.

It should be mentioned that in a closed system formed by a solid and a liquid, when the ratios of their equilibrium moisture contents depend on temperature, moisture transfer occurs even without an internal heat source. Thus, the initial equilibrium moisture content for cardboard and oil at t_1 = + 20°C and $u_{e0} = 1.0 \cdot 10^{-5}$ kg/kg leads with heating of the system to t_2 = + 50°C to the following moisture contents.

If there is no internal heat source in the cardboard, the oil moisture content is $2.50 \cdot 10^{-5}$ when G_0/G_c = 1000, $4.50 \cdot 10^{-5}$ when G_0/G_c = 100, and $5.15 \cdot 10^{-5}$ when G_0/G_c = 10. If there is a heat source with T_y = 1°C, the oil moisture content when G_0/G_c = 1000, 100 and 10 is, respectively, $5.67 \cdot 10^{-5}$, $7.07 \cdot 10^{-5}$ and $7.34 \cdot 10^{-5}$.

Therefore, both factors – difference in absorption characteristics of solid and liquid and internal temperature-dependent heat source – determine the moisture transfer in a closed system. The amount of moisture transfer depends on the range of temperature variation, the power of the internal heat source, the nature of its dependence on temperature and moisture content, and the absorption characteristics.

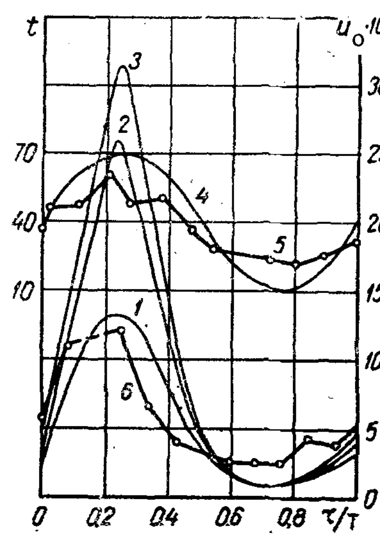

Fig. 4. Periodic variation of the moisture content of oil for G_0/G_c = = 1000 (1), 100 (2) and 10 (3) at t = 40 + 30 sin (τ/T) (4) and annual variation of daily mean temperature of upper layers of oil (5) and its moisture content (6) in a transformer according to [3].

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Lenin Power Engineering Institute, Ivanov